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Mixed-Integer Nonlinear Programming (MINLP) for Production Optimisation of Naturally Flowing and Artificial Lift Wells with Routing Constraints

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ABSTRACT

Real-time decision making by production engineers in a petroleum field can be very challenging, especially when multiple wells with diverse operating conditions and production behaviours are present. Hence, semi-analytic or heuristic procedures are unlikely to yield an optimal operating strategy. This paper implements a Real-Time Production Optimisation (RTPO) approach to maximising production from naturally flowing, gas-lifted and Electrical Submersible Pump (ESP)-assisted wells while satisfying multiple operational constraints. This is achieved via the application of reduced order models which are developed by querying a black box production network simulator multiple times using different inputs. Also exploited in this work is the inherent decomposable property of the production network, into smaller components (wells, valves pipelines and separators), such that mass balance equations comprise the algebraic constraints of the optimisation framework which is solved as an MINLP. The adopted formulation also offers the advantage of flexibility for problem adjustment under different practical operating conditions which are presented as case studies. The changes incorporated into the production system include: increased liquid handling capacity of downstream separators, decreased well productivity/increased water cut and well intervention problems. The ability of the adopted framework to provide accurate and speedy computations of the optimal production scenario makes it reliable for real-time decision support.

Keywords: Real Time Production Optimisation (RTPO); Mixed Integer Nonlinear Programming (MINLP); Well routing; Superstructure; Electrical Submersible Pumps (ESP); Gas Lift (GL); Naturally Flowing (NF); Gas Oil Ratio (GOR); Water Cut (WC)

1. Introduction

Maximizing recovery from proven reserves is a highly demanding task which requires consistent and rigorous application of modelling, simulation and optimisation tools by engineers in the petroleum industry (Gerogiorgis et al., 2009; Tavallali et al., 2013; 2014; Epelle and Gerogiorgis, 2017). Accurate understanding of physical flow phenomena, advanced mathematical techniques and high performance computing are important components embedded in these tools which have led to lower operational cost and increased process efficiency when they are systematically applied (Epelle and Gerogiorgis, 2018a). However, with the ever-increasing petroleum exploration difficulties faced by most companies, there is a commensurate need for the development of novel modelling methods, and also better integration strategies of simulation and optimisation techniques to increase field profitability (Codas et al., 2015; Gupta and Grossmann 2012a; 2012b). Although many high-fidelity simulation packages exist, it is essential that optimisation is considered in the early stages of design and model development by production engineers (Eason, 2018). A typical oil and gas production system is a collection of interconnected components which include: reservoirs, wells (with an attached surface choke), manifolds, and pipelines for routing the fluids to a separator where the respective phases (oil, gas and water) are split. Given the decomposable nature of the network, an optimisation framework that implements component-based simulation will significantly improve the efficiency of the entire system. Each component behaviour can be approximated by a simple algebraic relationship, which is a function of the system's properties (flow rates of respective phases, pressures and liquid and gas capacities). So far steady state simulation (over a short-term horizon) is the prevalent condition, these algebraic expressions are relatively uncomplicated (Ursin-Holm et al., 2014).

High-fidelity simulators employed in the petroleum industry can be very complex due to size, the type of physical phenomena modelled and accompanying model uncertainty; thus, the runtime for these simulators can be enormous especially if high accuracy is required (Epelle and Gerogiorgis, 2018b; 2018c). The number of network components and hence the number of equations describing each component, their physical interactions and interdependencies further imply that numerous equations and unknown variables are necessary to characterise the entire production system; this also adds to the complexity. The challenges faced from an optimisation viewpoint can be related to the type of problems solved (LP, MILP, NLP or MINLP) by available robust algorithms (commercial and open source). They can be classified based on the presence and type of constraints (equality and inequality), the amount of information provided by the model (derivative-free, first-order, second-order), the presence of nonlinearities and non-convexities, and the presence of discrete decisions in the problem (Kosmidis et al., 2004; 2005; Codas et al., 2012; Gunnerud et al., 2012). Leveraging the power of simulation within an optimisation framework often requires the development of equation-based approximations (surrogate or proxy models – polynomial interpolation, kriging, neural networks etc.) from the outputs of black-box simulators. The development of accurate proxy models is an active area of research that has received contributions from statistics, machine learning and engineering (Eason, 2018). When these models are to be used for optimisation purposes, the functional form and validation procedures are important concepts to consider in their construction.

In this work, it is demonstrated that an independent application of network simulation of an operating field does not yield the best possible improvement in oil production. Rather, a methodical application of robust optimisation methods with simulation guarantees process enhancement. This is achieved by developing explicit surrogate models in combination with well routing constraints which are compatible with the adopted optimisation algorithms; thus resulting in an MINLP formulation. In Section 2, a discussion of related publications on petroleum production optimisation is presented in detail, after which insights into the simulation and Real-Time Production Optimisation (RTPO) methodology are explained alongside the mathematical formulation of the optimisation problem. Findings based on the implemented case studies are presented subsequently with some conclusions derived.

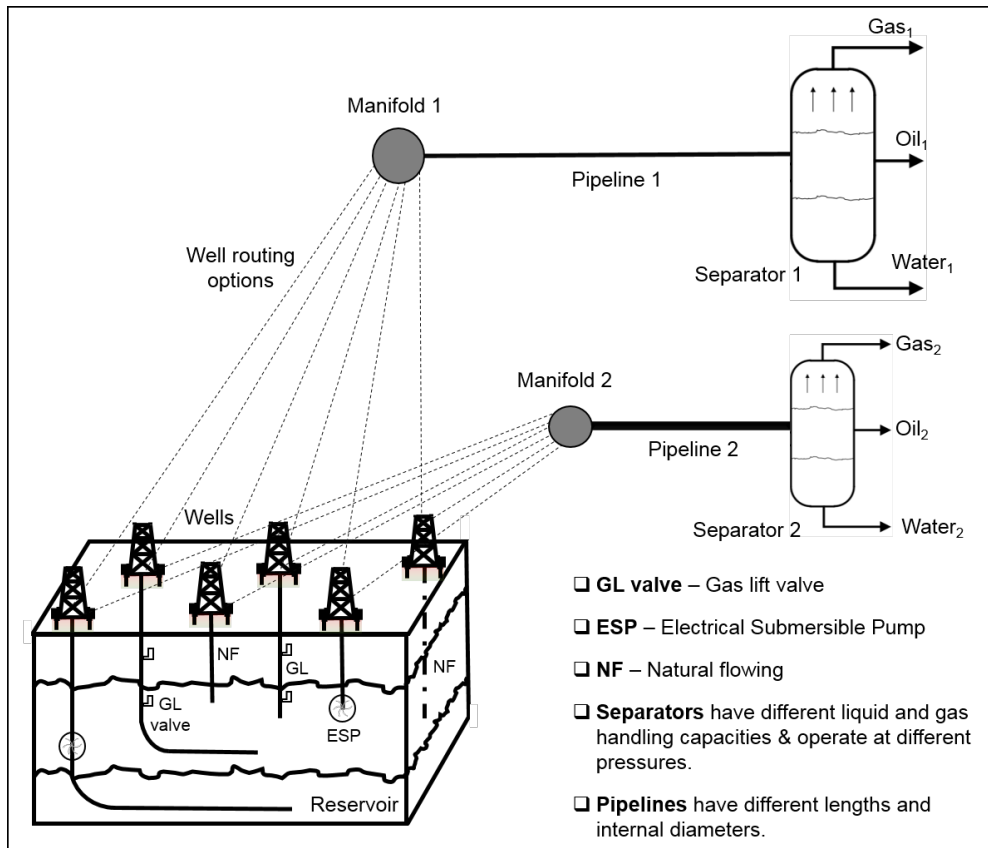


Figure 1: Petroleum production network structure and key elements.

2. Relevant Literature

Production systems optimisation in the oil and gas industry with practical operational constraints has received significant attention, documented in several publications. A comprehensive review by Khor and Elkamel (2007) classifies previous research endeavours into simulation-based, heuristic-based (Redden et al. 1974; Weiss et al. 1990; Litvak et al. 1995; 1997; 2002, such as choke diameter reduction and incremental GOR methods) and mathematical programming methods (Kosmidis, et al., 2005; 2005). They pointed out that these methods have generally addressed problems such as: design and operation of production systems, rate allocation, reservoir planning and development. However, the prevalence of naturally flowing vertical wells coupled with the application of structure-specific methodologies that are difficult to automate (when the production network becomes larger) limits their applicability to dynamic practical operations. Fortunately, increasing computational power over the past decade has resulted in more advanced optimisation algorithms, which are capable of simultaneously handling many crucial constraints and incorporating real field dynamic data to ensure operational feasibility (Barragán-Hernández et al., 2005; van Essen et al., 2011; Bellout et al., 2012; Hassan et al., 2013; Silva and Camponogara, 2014; Gu and Hoo, 2015; Siddhamshetty and Kwon, 2018; 2019). These advancements, which are incorporated in the current work, efficiently tackle the earlier-outlined automation and versatility challenges.

More recently, Tavallali et al. (2016) thoroughly evaluated the differences in research contributions (relating to production optimisation) from both petroleum and process systems engineering perspectives. In doing this, they grouped research endeavours from both perspectives into 3 main classes: oil field design, oil field operations and integrated field design and operations. In their discussion of this broad classification, subcategory problems such as rig scheduling (Iyer and Grossmann, 1986; van den Heever and Grossmann, 2000a; 2000b; 2001), flow scheduling (Kosmidis, 2005; Gunnerud et al., 2012), field planning (Gupta and Grossmann, 2012b; Tavallali et al., 2014; 2015; Humphries and Hayes, 2015), surface network design and well placement (Wang et al., 2012; Li et al., 2013) were also discussed. An extensive review of current advances and the applicability of several simulation packages and optimisation solvers was also presented. A major highlight from this review was the fact that contributions from the petroleum engineering community have primarily focused on

the subsurface, whereas, the process systems engineering community has paid more attention to the surface networks. Given the highly interconnected nature of a production system, more work is needed to address the challenges of integrated field design and operation; thus capturing surface and subsurface complexities.

Decline Curve analysis (based on real production data) has also been combined with production optimisation for fast prediction of future operating rates in the work of Kritsadativud et al. (2015). The motivation for this approach was to eliminate the huge time requirements in developing detailed numerical reservoir models, and the difficulty of using such models for optimisation purposes. Although their method does not adequately capture the underlying flow physics, they argue that it could provide good initial solutions for subsequent large-scale optimisation problems with full reservoir simulation. In spite of the prevalent application of Electrical Submersible Pumps–(ESP) artificial lift systems in petroleum field operations, production optimisation studies, which consider these well types, are very scarce. To the best of our knowledge, only the work of Hoffmann and Stanko (2016) addresses this kind of wells in the context of production optimisation. They formulated a Mixed Integer Linear Program (MILP via piecewise linearization), which determines the ESP performance characteristics for the different wells in their production network. The fast solution times they reported, demonstrated the real-time applicability of their formulation. However, representing the exact model characteristics via nonlinear constraints is an important attribute of this work, hence the formulation of a Mixed-Integer Nonlinear Program (MINLP). MINLPs combine the modelling capabilities of mixed-integer programs and nonlinear programming (NLP) into a flexible and multifaceted framework (Kronqvist et al., 2018). Besides the capability of such framework to model discrete decisions, the linear and nonlinear function handling ability enables accurate modelling of challenging and diverse phenomena. Despite this advantage, MINLP problems are very difficult to solve because they integrate all the complexities of their subclasses: the combinatorial nature of mixed integer problems and the difficulty of solving highly nonconvex nonlinear programs (Bussieck and Pruessner, 2003).

Gunnerud et al. (2013) elucidated the challenges of embedding a simulator in an optimization formulation. Simulators (depending on complexity) could be viewed as functions whose explicit forms are unknown, but compute outputs based on some input parameters. Furthermore, the inability of most black-box simulators to compute gradients necessary for speedy performance of an optimisation algorithm is an additional difficulty. In a bid to address these problems, they proposed a simulation-based optimisation method that incorporates the complex behaviour of production system components via simulator data approximation. This was based on a trust region approach coupled with an MINLP formulation. They demonstrated superior performance of their approach (in terms of solution quality and runtime) to a standard industry approach where derivative-free optimisation methods (which directly call the blackbox simulator at each iteration) are used. However, their approach requires in-depth knowledge of the simulator especially when large production networks with complicated nonlinear behaviours of the components are to be optimised.

Existing research contributions can be categorised into the type of problem solved (with or without geological uncertainty), time horizon involved, resulting optimisation formulation, optimisation algorithms/solvers and implementation platform/computer specifications employed, wellbore geometry used, the field production mechanism and the number of production components involved (size of problem). Novel contributions in this field can either focus on improvements related to these categories or the integration of other physical concepts and modelling tools. More recently, streamline simulation (Thiele and Batycky, 2006) is an important tool that has been adopted in production optimisation studies, which consider secondary production via water injection (Al-Zawawi et al., 2011; Azamipour et al., 2017; 2018; Epelle and Gerogiorgis, 2019). Taking into account this classification, there are four major novel elements in this paper, which are addressed in comparison to previous work discussed:

1. The combination of naturally flowing and artificially lifted (gas lift and ESP) wells creates complex pressure responses at the pipeline level which are accounted for via routing constraints and embedded in a complex economic objective function.
2. Key attention is paid to the bottomhole pressure of the well which in turn is affected by the wellhead pressure at a certain production rate. This is done in order to avoid sand production, which could be

detrimental to the overall system performance. The works of Tiffin et al. (2003), Wong et al. (2005) and Karantinos et al. (2017) provide some insights into downhole pressure control for the mitigation of sand production using field data, laboratory tests and mathematical modelling respectively.

3. The adaptability and flexibility of the proposed optimisation formulation to varying scenarios and practical operational difficulties is demonstrated.
4. The complexities of varying wellbore geometries with different multiphase flow properties are accounted for in the network model development.

3. Methodology

The proposed methodology is presented in two parts. The first part outlines the modelling and design considerations when creating the surface network and its components; the second explains the detailed problem formulation for which mathematical optimisation is applied. A real-time optimisation scenario is considered here; thus, a detailed reservoir model which captures the slow-paced dynamic reservoir behaviour, fluid properties, and production pressures and rates is not necessary.

3.1. Design and simulation considerations

3.1.1 Naturally flowing wells

Standard modelling procedures (casing, tubing and perforation design) are adopted in modelling the behaviour of naturally flowing wells in a multiphase flow simulator (PIPESIM[®] v2017.2). Robust multiphase flow correlations are employed for the pressure drop determination in the well tubing (Vertical Lift Performance curves–VLP); based on the well geometry and completion properties. Inflow Performance Relationships (IPR) are generated and used together with the VLPs to obtain the wells' operating points in the multiphase flow simulator. These curves essentially relate the multiphase flow rates in the wellbore to the bottomhole pressure and well head pressures. In order to obtain the pressure-rate response of a well, a nodal analysis (Fig. 2a) is run at different wellhead pressures. The obtained results can be approximated as an algebraic function which constitutes the constraints of the optimisation formulation. A similar procedure is adopted for the GL and ESP wells, but with extra nodal parameters such as the injection gas rate and the pump frequency. A detailed description of modelling considerations for the artificially lifted wells is presented next.

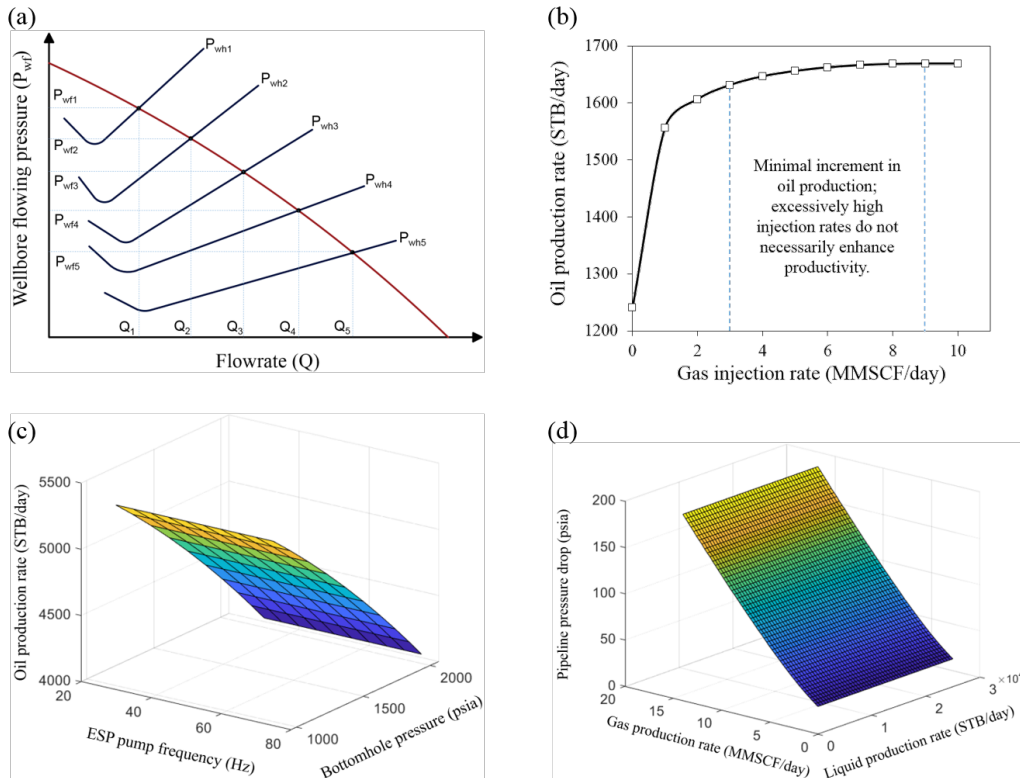


Figure 2: Typical nodal analysis (a) and gas lift optimisation curves (b); proxy model plots for an ESP well (c) and a pipeline (d).

3.1.2 Gas lift wells

In continuous gas lift (adopted here), a certain amount of gas at high pressure is introduced to aerate the fluid column (the tubing) so that the fluids readily flow to the surface (due to lower hydrostatic pressure). In order to perform this operation efficiently, it is often desired that injection is done via a single valve at the deepest possible point (this depends on the available surface injection pressure). In designing the gas lift system in the wellbore simulator, careful consideration was made in ensuring this single point injection scenario (Mukherjee and Economides, 1991; Gerogiorgis et al., 2006; Gerogiorgis and Pistikopoulos, 2008; Guo, 2011; Epelle and Gerogiorgis, 2019). Based on various lift gas availabilities, a production system analysis is necessary to ascertain well operating points using the bottom hole as the principal node of analysis. In performing this analysis, it was important to determine the injection rate that was sufficient to enable liquid production and to avoid excessive injection which prevents liquid flow due to pipe friction produced by the gas. Furthermore, excessive gas injection can significantly increase the gas capacity-handling requirement with minimal increment in oil production (Fig. 2b).

For a field-scale evaluation, the optimisation framework is designed to determine the optimal lift gas allocation to the respective wells based on their productivities and the total available gas for injection. Liquid fall-back during the unloading process of gas lift operations (especially intermittent gas lift) induces a back pressure effect on the formation (Guo, 2011). This could cause erratic rates and also impact flow rates of other wells close by (well flow interdependence), which could be detrimental to the flowline shared by these wells. The continuous gas lift operation adopted here significantly minimises this effect. Since the wells are connected to downstream separators operating at a specified pressure, any possible pressure fluctuations will be more pronounced in the wellbore than in the pipeline network (Kritsadativud et al., 2015). Furthermore, the short term horizon considered here implies that the reservoir and fluid properties do not vary significantly. Thus, back pressure effects at the surface pipeline can be considered negligible. However, the pressure drop along flowlines (between junctions $J-i$ and manifolds $M-i$ in Fig. 3) is assumed negligible and ignored in the optimisation computations.

3.1.3 Electrical Submersible Pumps (ESP)-assisted wells

The application of ESPs is particularly favourable for lifting high liquid volumes from wellbores with high productivity. Based on the desired volumetric flow rate of the well and the wellbore depth, the pump specifications (power, frequency and number of stages) can be calculated using the wellbore simulator. In order to avoid pump cavitation due to excessive free gas produced, high efficiency downhole separation is employed in the ESP design model. Since pump performance curves are based on water systems, a viscosity correction is implemented to account for the oil phase. Sand production is another important factor influencing the ESP performance; thus, it was important to estimate the critical drawdown pressure for limiting the liquid production based on the nodal analysis plots of the well. With the available pump manufacturer specifications, it was essential to ensure that the tubing size (internal diameter) selected could accommodate the outside diameter of the ESP with enough downhole clearance for the pump's liquid intake. This enabled accurate determination of the Total Dynamic Head (TDH) of the pump. ESP frequency was chosen as the main influencing parameter on the production capacities of the ESP wells. The power requirement of the pumps can be calculated subsequently from the optimal frequency and wellhead pressure of an ESP-assisted well. Incorporating constraints on the power requirements of the ESP was not necessary, because careful selection of high efficiency pumps based on the manufacturer's specifications characterised the ESP design process.

3.1.4 Other network components

The internal diameter, thickness, length and elevation difference were the pipeline specifications required for accurate pressure drop calculations. However, considerable effort was necessary for data generation at different operating conditions in the simulator. Based on the gas oil ratio (GOR), water cut (WC) and liquid rate (LR) ranges for the respective wells, a system analysis was performed multiple times to obtain high resolution data tables which were used for proxy model development and verification. In generating the proxy models, 25 data points are utilised for each well and 60 data points for each pipeline.

It is worth mentioning that the choke flow model is based on PIPESIM's mechanistic correlation that calculates the pressure drop across the choke using a weighted average of the liquid and gas phase pressure drops. The liquid and gas phase pressure drops are based on the Bernoulli's equation. The critical pressure ratio is calculated using the Ashford-Pierce (1975) equation; this distinguishes subcritical from supercritical flow. However, the latter (supercritical flow conditions) represents a situation that rarely occurs in reality (PIPESIM, 2017); and does not manifest in our simulations. The choke bean size is a constant value in the simulations performed and is initially assumed 100% open. Binary variables are introduced to route production from wells only. In the case that a well violates the separator capacity or water capacity constraints, then the algorithm automatically shuts the well. Besides the binary variables present ($x_{w,g,inj}$, $x_{w,l,ESP}$, $x_{w,p}$), all other variables are continuous.

In generating the proxy model, the Hagedorn and Brown (1965) correlation is adopted for the vertical multiphase flow; whereas the revised Beggs and Brill (1973) correlation is utilised for horizontal multiphase flow calculations. The basic assumption for the friction model (Moody, 1944) is that the pressure drop during transient flowing conditions is the same as the steady flowing conditions using an average instantaneous transient velocity and the apparent mixture properties.

3.2. Problem definition and optimisation formulation

Given the network superstructure (Fig. 3), comprising of a single reservoir, 6 wells (3 pairs of NF, GL & ESP wells), 2 manifolds, 2 pipelines and 2 separators, the aim is to optimise the Net Present Value (NPV) by determining the optimal well controls, lift gas allocation and routing strategy on a real-time basis. Operational constraints include the wellbore- and pipeline-approximated models, mass and energy balances across the network, upper and lower bounds on all operating pressures and flow rates (for the avoidance of sand production). In the mathematical description, wells are assigned the index w , manifolds, m , pipelines, p , separators, s , oil, water and gas phases, o , wat , g , liquid phase l ; collectively all phases are represented as i .

$$\text{Max } (NPV) = ROP + RGP - CWP - CQ_{g,inj} - CQ_{l,ESP} \quad (1)$$

The objective function (Eqs. 1-6) maximises the Net Present Value (NPV) of the production system while ensuring the wellbore pressures (wellhead, wh and bottomhole, wf) are within acceptable ranges that prevent sand production (Eq. 7-11). The cost indices r_o , r_g , r_{wat} , $r_{g,inj}$, and $r_{l,ESP}$ were \$70/STB, \$2000/MMSCF, \$20/STB, \$10,000/MMSCF, and \$12/STB respectively. The total revenue from produced gas produced (Eq. 3) excludes the quantity of injected gas in the GL wells. Well flow behaviour is approximated via the algebraic relationships (Eqs. 12, 13 and 15) for the NF, GL and ESP wells respectively. Eq. 14 ensures that the allocated lift gas to the GL wells is below the field available gas for injection. Binary variables $x_{w,p}$ assigned to each well ensure that the produced fluids from a well are routed to one of the pipelines. Eq. 16 represents the well choke settings which ensure that if a well is routed to a particular pipeline, then the manifold pressure must be lower than the wellhead pressure of the well to avoid backward flow of material. The mass balance constraint between wells and pipelines is represented as Eq. 17. In the formulation, it is assumed that the separators operate at a fixed known pressure, P^s ; thus, Eqs. 19 and 20 ensure that the manifold pressure, P^m , is sufficient to overcome the pressure drop in the pipelines, and the fluids eventually reach the separator at the desire pressure. Liquid and gas capacity constraints of the separators are represented by Eqs. 21 and 22 respectively.

$$ROP = r_o \times \sum_{w=1}^{Nprod} q_o \quad (2) \quad q_{i,w,GL} = f(P_w^{wh}, Q_{w,ginj}) \quad \forall i, \forall w \quad (13)$$

$$RGP = r_g \times \sum_{w=1}^{Nprod} q_g \quad (3) \quad Q_{w,ginj} \leq Q_{w,ginj}^{max} \quad (14)$$

$$CWP = r_{wat} \times \sum_{w=1}^{Nprod} q_w \quad (4) \quad q_{i,w,ESP} = f(P_w^{wh}, f_{i,w,ESP}) \quad \forall i, \forall w \quad (15)$$

$$CQ_{g,inj} = r_{g,inj} \times \sum_{w=1}^{Ng,inj} [\sum x_{w,g,inj} \times q_{g,inj}] \quad (5) \quad x_{w,p} P^m \leq P_w^{wh} \quad \forall w, \forall p \quad (16)$$

$$CQ_{l,ESP} = r_{l,ESP} \times \sum_{w=1}^{Nl,ESP} [\sum x_{w,l,ESP} \times q_{l,ESP}] \quad (6) \quad Q_{i,p} = \sum_w (x_{w,p} \times q_{i,w}) \quad \forall i, \forall p \quad (17)$$

$$P_w^{wf} = f(P_w^{wh}, D_{choke}) \quad \forall NF \quad w \quad (7) \quad \sum_p x_{w,p} \leq 1 \quad (18)$$

$$P_w^{wf} = f(P_w^{wh}, Q_{g,inj}) \quad \forall GL \quad w \quad (8) \quad \Delta P = f(q_{p,o}, q_{p,wat}, q_{p,g}) \quad (19)$$

$$P_w^{wf} = f(P_w^{wh}, f_{ESP}) \quad \forall ESP \quad w \quad (9) \quad P^s = P^m - \Delta P \quad (20)$$

$$P_{w,min}^{wf} \leq P_w^{wf} \leq P_{w,max}^{wf} \quad \forall w \quad (10) \quad \sum_p q_{g,p} \leq CG^s \quad (21)$$

$$P_{w,min}^{wh} \leq P_w^{wh} \leq P_{w,max}^{wh} \quad \forall w \quad (11) \quad \sum_p q_{l,p} \leq CL^s \quad (22)$$

$$q_{i,w,NF} = f(P_w^{wh}, D_{choke}) \quad \forall i, \forall w \quad (12)$$

The proxy model generation (Figs. 2a- 2d) using least-squares method and the optimisation formulation (objective and constraint functions) were written in MATLAB and solved using the Basic Open-source Nonlinear Mixed Integer Programming (BONMIN) solver via the Opti toolbox interfacing platform (Currie and Wilson, 2012). The ‘B-BB’ and ‘B-OA’ algorithms of BONMIN were utilised in finding good local solutions. The former (B-BB), implements a simple branch and bound algorithm based on the solution of the continuous NLPs at each node of the search tree and subsequently branching on the integer variables. This is made possible by modifying Cbc (a mixed integer linear programming solver) so that LP solutions at each node of the tree are replaced by NLP solutions (Bonami et al., 2008). NLP solutions at each node are obtained speedily using the IPOPT solver. This B-BB algorithm of BONMIN is similar to the one implemented in the solver, ‘SBB’. The latter (B-OA), is an outer-approximation branch and cut algorithm, similar to that implemented in DICOPT. It iteratively solves and improves the MIP relaxation of the MINLP problem and also solves the NLP subproblems (Fletcher and Leyffer, 1994; Gupta et al., 1985). In the algorithm, a single tree search is performed, and the resulting NLP solutions are used to progressively tighten the MILP relaxation. The motivation of this approach was to avoid the sequential solution of several relaxed MILPs; thus increasing the computational speed (Bonami et al., 2008).

Successful implementation of the described formulation (Eqs. 1-22) requires that proxy models are developed within a certain range of wellhead and bottomhole pressures in which these models accurately approximate the simulator outputs; it is preferable that this range is not very far from the initial guesses. From preliminary tests performed, the optimisation algorithm fails after a certain number of iterations when the input range (search space involved) is very large or the upper and lower bounds of the independent decision variables are very loose. Thus, it is imperative that reasonably tight bounds are set and that the parameters of the proxy model are updated via rerunning the black box simulators; especially when operating conditions change significantly (e.g. a change in flow regime or well productivity). Gunnerud et al. (2013) provide recommendations/algorithmic modification strategies for updating the proxy models and their corresponding trust regions.

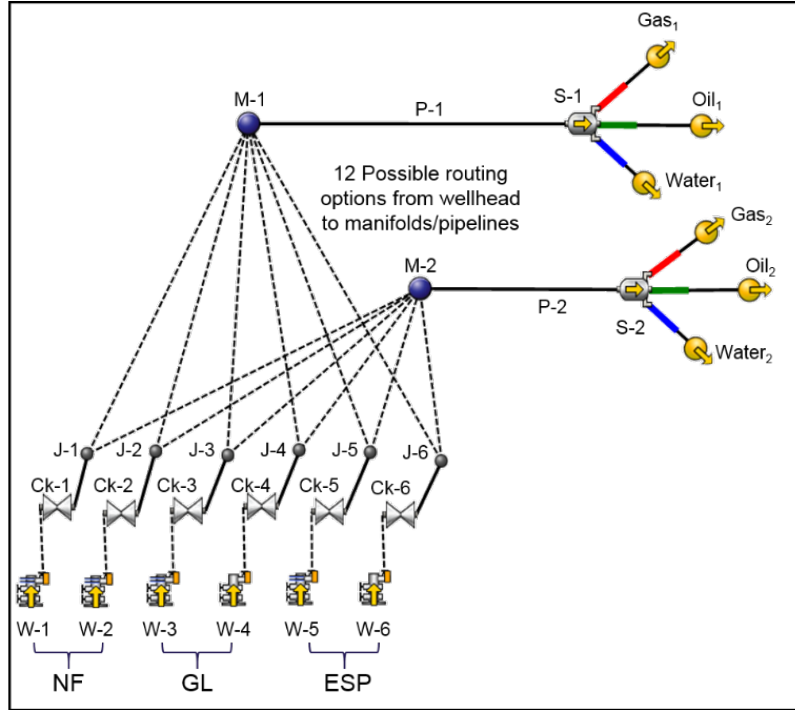


Figure 3: Superstructure of network connections for the petroleum production system.

4. Results and Discussion

The superstructure of the production network with all possible connections is shown in Fig. 3; 12 possible routing options exist for which the optimisation algorithm is expected to find the best well-manifold connections that guarantee an optimal NPV.

Furthermore, three case scenarios are compared to a base case in which all wells perform well with high productivity indexes (>1 STB/day.psi), relatively low average water cut (30%), an average GOR of 800 SCF/STB and limited water and gas handling capacities. By considering these case studies, the flexibility of the optimisation formulation is demonstrated.

- Base Case (BC): Limited separator handling capacities.
- Case Study 1 (CS1): Increased liquid and gas handling capacities of S-2.
- Case Study 2 (CS2): Decreased well productivity and increased water cut for the NF well (W-1).
- Case Study 3 (CS3): Well intervention on the W-5 and W-6 due to ESP damage.
- Case Study 4 (CS4): Switching W-3 to ESP mode with increase separator handling capacities.

Table 1: Separator capacities and operating pressures for all cases explored.

Case study	Separator	Operating pressure (psia)	Liquid capacity (STB/day)	Gas Capacity (MMSCF/day)
Base Case (BC)	S-1	80	15000	9
	S-2	50	10000	6
Case Study 1 (CS1)	S-1	80	15000	9
	S-2	50	15000	9
Case Study 2 (CS2)	S-1	80	15000	9
	S-2	50	10000	6
Case Study 3 (CS3)	S-1	80	15000	9
	S-2	50	10000	12
Case Study 4 (CS4)	S-1	80	15000	10
	S-2	50	15000	10

The separator, well and pipeline characteristics are given in Tables 1 and 2 respectively. S-1 is connected to M-1 (Figs. 1 and 3) by a longer pipeline (but with a smaller internal diameter) compared to S-2. The well characteristics are very similar as shown in Table 2; however, their perforation intervals and permeabilities around the well are different, thus resulting in the varying production responses observed in Fig. 4.

Table 2: Reservoir, well and pipeline parameters

Parameter	W-1	W-2	W-3	W-4	W-5	W-6	P-1	P-2
Reservoir pressure (psia)	3800	3800	3800	3800	3800	3800	–	–
Well type	Deviated	Vertical	Deviated	Vertical	Deviated	Vertical	–	–
GOR (SCF/STB)	800	780	810	785	800	800	–	–
WC (%)	30	30	25	30	30	28	–	–
True Vertical Depth – TVD (ft)	9000	10000	9500	10000	9500	10000	–	–
BHP constraint to avoid sand production (psi)	700	700	700	700	700	700	–	–
Productivity Index (STB/day.psi)	2.5	1	2.3	1.5	2.8	2	–	–
TVD of gas lift valve (ft)	–	–	5800	7500	–	–	–	–
Assumed temperature along wellbore (°F)	200	200	200	200	200	200	200	200
Tubing diameter (in)	3.5	3.5	3.5	3.5	3.5	3.5	–	–
Pipeline length (ft)	–	–	–	–	–	–	6000	4000
Pipeline internal diameter (in)	–	–	–	–	–	–	10	12
Pipeline internal roughness (in)	–	–	–	–	–	–	0.001	0.001

Fig. 4 illustrates the production rates obtained via simulation with the multiphase simulator at a wellhead pressure of 380 psia, before the proposed optimisation formulation is applied. Based on the optimal wellhead pressures and flowrates, the optimal routing configurations are determined for the 5 different case scenarios and presented next.

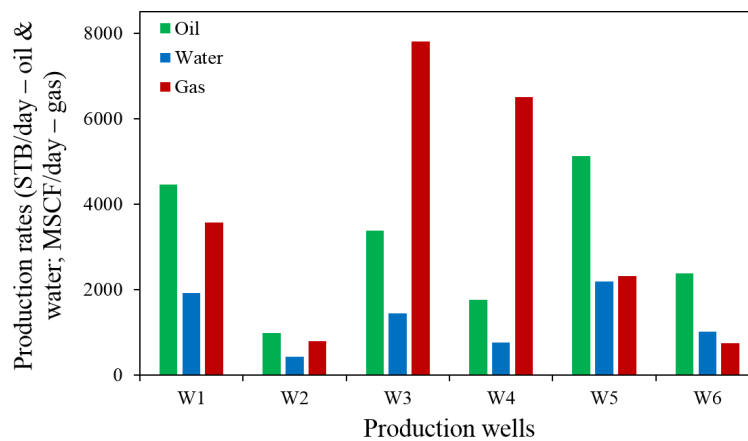


Figure 4: Simulated well production performance.

4.1 Base Case

It is illustrated in Fig. 5 that the optimal routing strategy involves connecting the GL well (W-4) and the ESP wells (W-5 and W-6) to M-1, and routing the NF wells to M2; whereas, W-3 is shut. A physical explanation for this routing strategy is that the gas handling capacity of the separators is relatively lower

than the combined gas production rates from W-3 and W-4. These high gas production rates can be attributed to the originally injected gas during the gas lift operation.

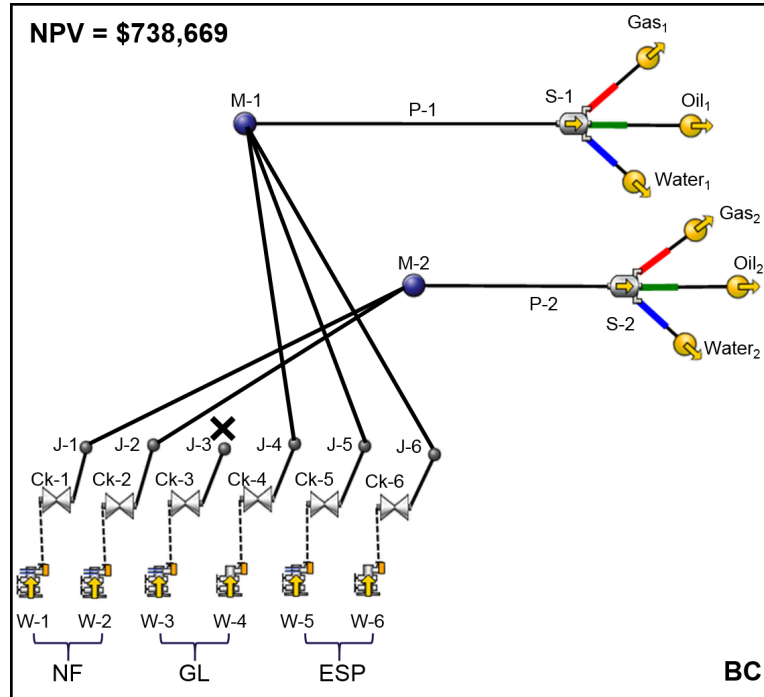


Figure 5: Optimal routing structure for the Base Case

For W-3 to be shut in place of W-4 (Fig. 5), it implies that the revenue due to additional oil production does not outweigh the cost of gas injection in this well (W-3). Conversely, W-4 remains open, despite its lower oil production rate, compared to W-3. It is also worth noting the higher water production rate of W-3 in comparison to W-4 (Fig. 4) has made it a less preferable candidate for improving the NPV. Typical heuristic approaches which might involve opening high oil producing wells, with little consideration to the gas and liquid capacities of the separating units would not guarantee net improvement in field profitability. Enhancing the performance of W-3 and W-4 could involve more strategic positioning of well perforations that would prevent gas and water coning, thus reducing the wells' GOR and WC. Furthermore, ESPs could be used in place of GL in these wells for GOR reduction.

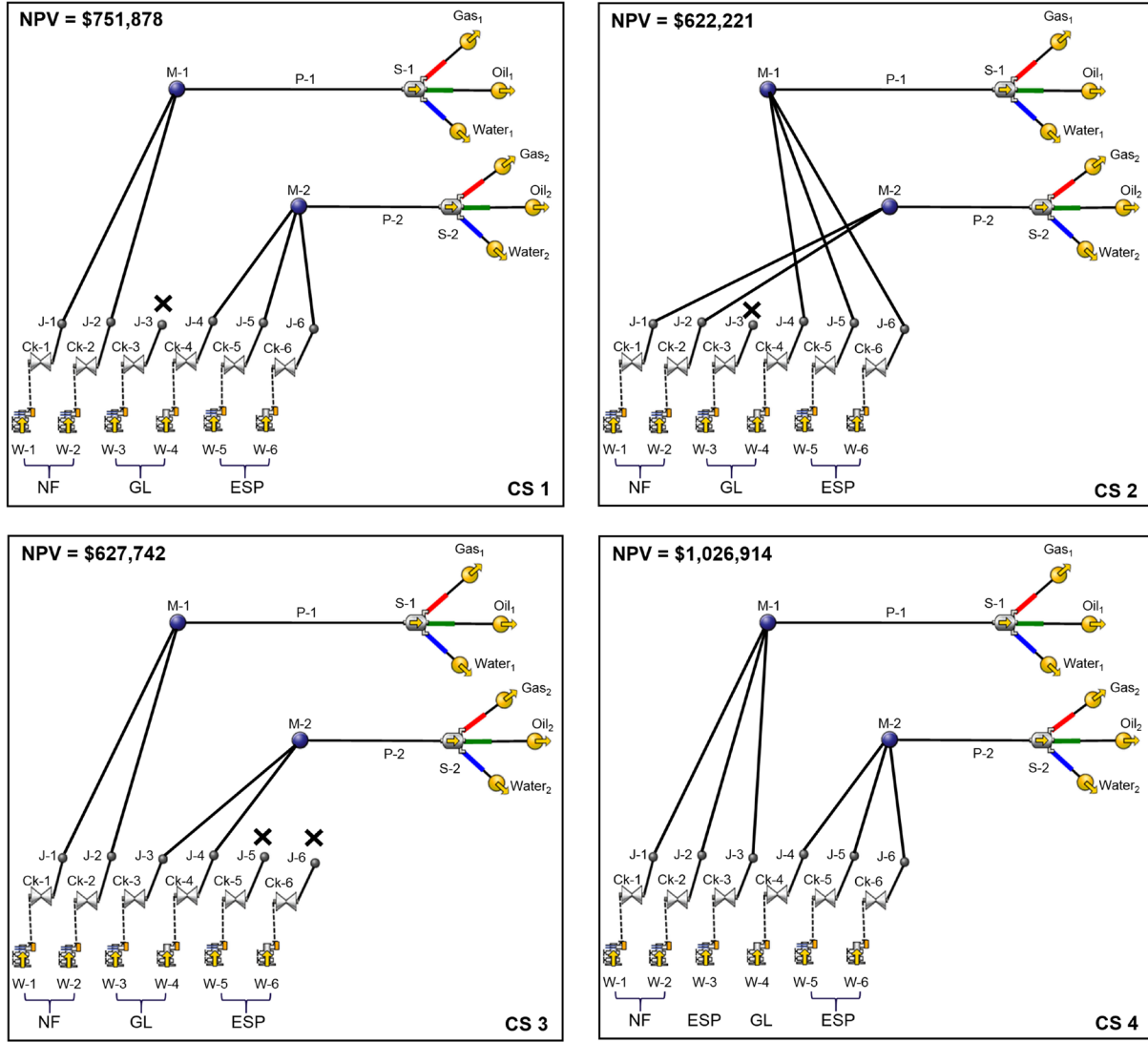


Figure 6: Optimal routing structures for all case studies (CS1, CS2, CS3 and CS4) explored.

4.2 Case Study 1 (Increased liquid and gas handling capacities of S-2)

Increasing the liquid and gas handling capacities of S-2 to 15,000 STB/day and 9 MMSCF/day (same as the capacities of S-1, Table 1), changes the routing structure. As observed in Fig. 6, there is an increase of 1.8% in the NPV (compared to the base case) due to capacity enlargement of S-2. Although the cost of this enlargement is not included in the optimisation formulation, this 1.8% improvement would cumulatively surpass the expansion costs over a long production horizon. It is observed in Fig. 6 that the shorter pipeline (P-2) with a lower operating separator pressure attached (S-2) is the preferred routing option for W-4, W-5 and W-6 respectively. Besides the increased capacity, another factor influencing this optimal routing option is the size and length of the pipeline (P-2); its shorter length and larger diameter implies that the pressure drop through the pipeline (P-2) is lower. Furthermore, the lower operating pressure of the separator (S-2) implies that a lower manifold pressure and in turn a reduced wellhead pressure would guarantee forward flow of the fluids from the wells. This reduced wellhead pressure translates to a higher production rate response from these wells. Due to capacity constraints, W-3 is still shut and W-1 and W-2 are routed to S-1 via P-1.

4.3 Case Study 2 (Decreased productivity and increased water cut of W-1)

In this case study, the reservoir permeability of the Joshi steady state IPR model (for the deviated well W-1) is reduced from 100 mD to 80 mD and the water cut of the well is increased from 30% to 45%. The operating gas and liquid handling capacities are same as that of the base case. Although there is an inevitable reduction in the NPV by 16% compared to the base case, the routing structure is maintained (same as the base case). Besides the capacity limitations earlier explained in Section 4.1, this structure

remains the optimal because by closing W-3, the pressure drop in either of the pipelines (that would have ensued if W-3 were connected to M-1 or M-2) is significantly reduced, and the production rate from other wells is consequently increased. Considering the intricacy and the number of parameters to simultaneously consider, heuristic methods cannot guarantee optimality of the routing structure of this production network. Additionally, it becomes impossible to apply such methods when the network becomes very large as there would be several routing possibilities.

4.4 Case Study 3 (Well intervention due to ESP damage)

In this illustrative example, a well intervention is carried out on the ESP assisted wells in order to perform maintenance activities and subsequently reinstall the ESPs. Hence W-5 and W-6 cannot produce and are considered closed wells by the optimisation solver. Furthermore the liquid and gas handling capacities of S-1 are retained at the same values as the base case; similarly, the liquid handling capacity of S-2 is maintained at the base case value. However, only the gas handling capacity of S-2 has been increased from 6 MMSCF/day to 12 MMSCF/day. The resulting routing configuration based on these modifications are shown in Fig. 6. It is observed that the high gas producing wells (W-3 and W-4) are preferably routed to the separator with the highest gas handling capacity; whereas the NF wells are preferably routed to S-1 with the lower gas handling capacity. Despite shutting the 2 ESP wells, the NPV of CS3 \$627,742 is comparable to that of CS2 (in which only 1 well is shut) with a \$622,221 NPV. On analysing the field production rates of all phases in Table 3, an explanation for this similarity can be derived. Although CS2 yields a 23% higher oil production rate, its water production rate and thus its production cost is significantly higher (50%) than that of CS3. Furthermore, the gas production rate of CS3 is 40% higher than CS2. The cumulative effect of these differences is a slight enhancement (0.9%) in the profitability of CS3 over CS2 as reflected in the NPV.

4.5 Case Study 4: Switching W-3 to ESP mode with increase separator handling capacities

Given the previously identified problems with W-3, its performance is enhanced by switching the artificial lift operation from gas lift to an ESP in this case study. This translates to an increase in the oil production rate of this well compared to the increment in oil production that was obtainable via gas lift operation. It is illustrated in Fig. 6 that the optimal status of the network is now to open W-3 and route it to M-1. The increased separator capacities compared to the base case also resulted in the opening of all wells; thus resulting in a 39% increase in the NPV compared to the base case. Despite the pressure drop differences in the pipeline, the similarity of separator capacities has allowed a correspondingly equal split in the produced fluids from the 6 wells (3 wells each). It can thus be inferred that the handling capacities of the produced fluids and the wells' mode of operation significantly influence the optimal routing strategy.

Table 3 also illustrates good utilization of liquid and gas storage capacities by the optimisation algorithm for all case studies. However, CS 4 ranks highest with 91% capacity usage for the liquid phase, whereas the base case ranks highest with 85% usage of the total gas capacity. It is worth mentioning that the separator capacities mentioned here do not necessarily imply the capacity of a single separating vessel but rather multi-stage separation/separators equipped with extra storage is also possible. Although detailed design of the separating vessels and capacities is not within the scope of this paper, the optimisation methodology applied here could aid production engineers in the choice of appropriate vessel capacities in order to avoid redundancy.

Table 3: Optimal field production and injection rates, ESP power requirements and routing strategy.

Field parameter		BC	CS1	CS2	CS3	CS4	
Oil production rate (STB/day)		14930	15209	13476	10924	19064	
Water production rate (STB/day)		6399	6518	7016	4682	8171	
Liquid production rate (STB/day)		21328	21726	20492	15605	27235	
<i>Available liquid capacity (STB/day)</i>		<i>25000</i>	<i>30000</i>	<i>25000</i>	<i>25000</i>	<i>30000</i>	
<i>Liquid capacity utilized (%)</i>		<i>85.3</i>	<i>72.4</i>	<i>82.0</i>	<i>62.4</i>	<i>90.8</i>	
Gas production rate (MMSCF/day)		12.81	12.91	11.64	16.34	16.00	
<i>Available gas capacity (MMSCF/day)</i>		<i>15.00</i>	<i>18.00</i>	<i>15.00</i>	<i>21.00</i>	<i>20.00</i>	
<i>Gas capacity utilized (%)</i>		<i>85.4</i>	<i>71.7</i>	<i>77.6</i>	<i>77.8</i>	<i>80.0</i>	
Total gas injection rate (MMSCF/day)		3.8	3.8	3.8	7.6	3.8	
Total ESP power requirement (hp)		520.3	574.8	522.1	–	702.5	
Optimal routing strategy of the surface production network for all case studies considered							
x _{1,1}	W-1	PIPELINE 1	0	1	0	1	1
x _{2,1}	W-2		0	1	0	1	1
x _{3,1}	W-3		0	0	0	0	1
x _{4,1}	W-4		1	0	1	0	0
x _{5,1}	W-5		1	0	1	0	0
x _{6,1}	W-6		1	0	1	0	0
x _{1,2}	W-1	PIPELINE 2	1	0	1	0	0
x _{2,2}	W-2		1	0	1	0	0
x _{3,2}	W-3		0	0	0	1	0
x _{4,2}	W-4		0	1	0	1	1
x _{5,2}	W-5		0	1	0	0	1
x _{6,2}	W-6		0	1	0	0	1

4.6 Solver performance analysis

It can be shown in Table 4 that optimal results are obtained in a relatively short time; although this is largely due to the problem size, decomposition techniques that enhance solution speed (such as the Dantzig-Wolfe and Lagrange decomposition within a reformulated MILP) can readily be applied to larger problems (Gunnerud and Foss, 2010). The run time required for the applied method in this paper demonstrates its applicability to real-time decision making in practical operations. A major influencing factor on the solution time required is the number of discrete variables present.

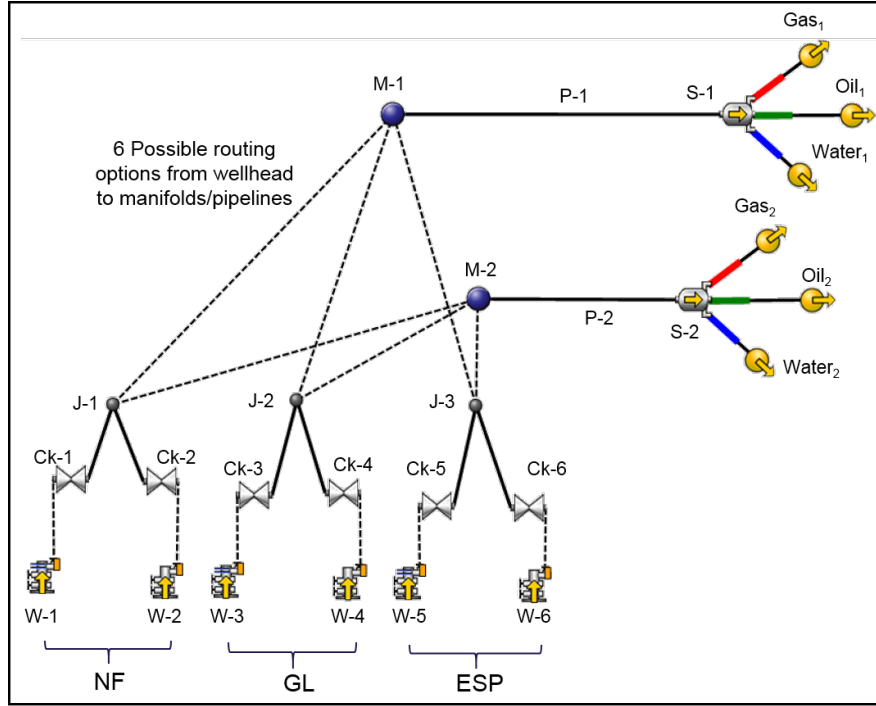


Figure 7: Network superstructure with reduced routing options.

It is observed in CS3 that, where the number of routing variables becomes 8 (compared to the base case in which 12 routing variables are present), the solution runtime reduces by 2 orders of magnitude (Table 4). In addition, the optimality gap and number of iterations are significantly reduced, thus indicating the relatively lower computational effort required for a good-quality solution. Hence, for very complex networks in which the number of wells become very large, a possible strategy for run time reduction is to group the production wells into clusters as shown in Fig. 7, so that the number of routing variables is reduced. Although this is usually done in practical operations (gathering wells at different clusters or junctions, J-1, J-2 and J-3 – Fig. 7), this approach further constrains the optimisation algorithm (and may yield suboptimal results) compared to a scenario in which the algorithm’s exploration space for better possible routing options is larger. However, it is vital to maintain a balance between solvability of the optimisation problem and the desired accuracy at all times. Moreover, in a situation where the well position/gathering network already exists/is fixed (which is the case in this paper), the existing structure has to be maintained and the routing options alone optimised. Infrastructural planning problems, which involve well placement decisions, can incorporate these different routing decisions (Figs. 3 and 7) as additional constraints.

Although the B-OA algorithm of BONMIN was able to provide good solutions in much faster time (<10 sec) compared to the B-BB algorithm in a few cases, it was desirable to progressively monitor the change in the objective value at each iteration which was only outputted by the B-BB algorithm. This provided some insight into troubleshooting an unsuccessful optimisation run (changing variable bounds, or initial guesses etc.). Furthermore, in the preliminary test cases run, the B-BB algorithm proved more stable, with higher NPVs obtained compared to the B-OA algorithm for the problem described herein. Considering the problem’s non-convexity, and the fact that no specific heuristic method for treating nonconvex problems is implemented within the OA framework (Bonami and Lee, 2013), the B-BB algorithm was adopted for all optimisation runs in this paper.

Several factors are responsible for the solver performance shown in Table 4. One of the steps taken to ensure stability of the solver and repeatability of optimal solutions upon several runs was to reduce the relative disparity in the magnitude of the different variables, particularly during the proxy model generation. Furthermore, obtaining accurate proxy models that represent the simulator output within the supplied input range is vital for good solver performance. Generated well proxy models had an average error of 0.5% whereas that for both pipelines was 0.9%. In both cases, the maximum error was less than 3%, thus demonstrating the structure/formulation quality of the proxy models.

Table 4: Solver performance analysis (BONMIN B-BB algorithm)

Case Study	Run time (sec)	Number of nodes	Number of iterations	Optimality Gap	NPV (\$) – MINLP solution	NPV (\$) – NLP solution	Absolute percentage difference (%)
BC	96	4	423	0.131	738,669	738,659	0.001
CS1	74	40	1346	0.197	751,878	751,878	0.00
CS2	81	6	709	0.232	622,221	633,910	1.90
CS3	0.9	2	23	0.003	627,742	627,742	0.00
CS4	102	8	4612	0.006	1,026,914	1,026,918	0.00

In order to investigate possible improvements in solution quality in terms of the NPV, the discrete variables obtained by solving the MINLP problem using BONMIN (shown in Table 3) are fixed and the resulting NLP problem is solved using IPOPT. This idea stems from the work of Gupta and Grossman (2012), in which they sought improvements (increased NPV) to their local MINLP solutions after fixing the optimal discrete solutions obtained. Table 4 also summarizes the differences in NPV obtained. It was discovered that this technique worked satisfactorily with a 1.9% increment in NPV for CS2. However, it resulted in a slightly reduced NPV for the Base Case. We attribute this occurrence to the fact that the NLP solver (IPOPT) is not a global solver by design and may have been trapped at a locally optimal solution. However, the percentage difference between the NPVs of the MINLP and NLP formulations of the base case are negligible.

Thus far, we have presented, formulated and analysed a unified computational approach which combines polynomial-based surrogate models and operational constraints that simultaneously accounts for chokes, pipelines and wellbore physics (of different types – NF, GL, ESP) in the same oil field. Although a direct comparison of heuristic and deterministic methodologies escapes the scope of this study, a summary of Kosmidis et al. (2005) computational evaluation of both methodologies is given in Fig. A2 of the Appendix. They realise a 14.1% improvement in oil production compared to the heuristic method that is based on choke diameter reduction and the incremental GOR concept.

5. Conclusions

In this paper an optimisation framework that simultaneously considers the production behaviours of naturally flowing, gas-lifted and ESP-assisted wells is proposed. Simulation and computational analyses based on algebraic proxy models were carried out considering a synthetic but practical production network. Compared to previous optimisation formulations, this work has implemented a more realistic objective function in determining the optimal operating conditions and routing configurations. Specifically, the optimal field power requirements for the respective ESPs and the optimal gas injection rates are determined.

Separator handling capacities of the respective gas and liquid phases is a highly influential factor on the optimal routing strategy. Considering the combination of several other contributing factors, such as the pipeline pressure drop and separator operating pressure, heuristic-based routing methodologies do not guarantee an optimal operating configuration.

Speedy computations of the resulting MINLP problem using robust MINLP algorithms ensures that solutions of the optimal routing strategy in a production network can be obtained in real-time. Expansion of the applied formulation to larger fields with more wells will likely yield solutions within short time periods provided a systematic model parameter update loop is embedded in the formulation. Although the time required for proxy model data generation is not included in the analysis, automated data generation capabilities are emerging attributes of high-fidelity simulators that can be exploited for further computational time reduction.

The proposed optimisation formulation demonstrates good utilisation of separator capacity for routing produced fluids. It is thus useful for production network design purposes, when decisions relating to the

size of separation facilities are to be made. Furthermore, its robustness is also illustrated by the similar NPV results obtained between the MINLP and NLP formulations (based on discrete solutions of the MINLP formulation).

The adaptability of the proposed formulation to changing operational conditions is also demonstrated via 5 different case studies. It was discovered that changing the artificial lift mechanism could result in a 39% improvement in the NPV.

Future investigations could consider the option of routing a well's fluids to more than one pipeline at a time. Furthermore, incorporating piping costs in the NPV objective function is an important extension of the current work that is worth investigating.

6. Acknowledgements

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7. Nomenclature

D_{choke}	Choke size (in)	$r_{l,ESP}$	Unit ESP liquid operating cost (\$/STB)
$FPSO$	Floating Production Storage and Offloading	N_{prod}	Number of production wells (-)
f_{ESP}	ESP operating frequency (Hz)	$N_{g,inj}$	Number of gas lift wells (-)
GOR	Gas Oil Ratio (SCF/STB)	$N_{l,ESP}$	Number of ESP-assisted wells (-)
k	Permeability (mD)	i	Fluid phase index (-)
$MSCF$	Mega standard cubic feet	m	Manifold index (-)
NPV	Net Present Value (\$)	o	Oil phase index (-)
ΔP	Pressure drop in pipeline (psi)	p	Pipeline index (-)
PI	Well productivity index (STB/day.psi)	w	Production well index (-)
P	Pressure (psia)	wat	water phase index (-)
P^m	Manifold pressure (psia)	x_w	Binary routing variable (-)
P^s	Separator pressure (psia)	Cbc	Coin-or branch and cut
P_r	Reservoir pressure (psia)	CWP	Cost of water production (\$)
P_{wf}	Bottomhole flowing pressure (psia)	$CQ_{g,inj}$	Cost of gas lift operation (\$)
P^{wh}	Wellhead pressure (psia)	$CQ_{l,ESP}$	Cost of ESP operation (\$)
CG^s	Separator gas capacity (MSCF/day)	ESP	Electrical Submersible Pump (-)
CL^s	Separator liquid capacity (STB/day)	GL	Gas lift (-)
$Q_{g,inj}$	GL well gas injection rate (MMSCF/day)	GOR	Gas oil ratio (SCF/STB)
$Q_{l,ESP}$	ESP well liquid production rate (STB/day)	NF	Natural flowing (-)
Q, q	Flowrates (STB/day or MMSCF/day)	ROP	Revenue from oil production (\$)
r_{op}	Unit oil price (\$/STB)	RGP	Revenue from gas production (\$)
r_{gp}	Unit gas price (\$/MSCF)	WC	Water cut (%)
r_{wp}	Unit water production cost (\$/STB)	TVD	True Vertical Depth (ft)
r_{ginj}	Unit gas injection cost (\$/MMSCF)		

Appendix

Model parameters

The parameters in Table A1 represent the input data to the surface network simulator for accurate fluid description through the wells, chokes, flowlines, manifolds, pipelines and separators respectively.

Table A1: Parameters used in the surface network model

PVT model	Black oil model
Gas-oil ratio (SCF/STB)	720-800
Water cut (%)	20-30
Oil specific gravity (API)	45
Gas density (lbm/ft ³)	0.0507
Bubble point pressure (psi)	1500
Pipeline temperature (°F)	100

Proxy models and validation

The functional form of the proxy models used in this work are shown in Eqs. A1-A4 respectively

$$Q_{o,NF} = \alpha_0 + \alpha_1 P_{wh} + \alpha_2 P_{wh}^2 \quad (A1)$$

$$Q_{o,GL} = \beta_0 + \beta_1 P_{wh} + \beta_2 Q_{g,inj} + \beta_3 P_{wh}^2 + \beta_4 Q_{g,inj}^2 + \beta_5 P_{wh} Q_{g,inj} \quad (A2)$$

$$Q_{o,ESP} = \delta_0 + \delta_1 P_{wh} + \delta_2 f_{ESP} + \delta_3 P_{wh}^2 + \delta_4 f_{ESP}^2 + \delta_5 P_{wh} f_{ESP} \quad (A3)$$

$$P_l = \varepsilon_0 + \varepsilon_1 Q_{lg} + \varepsilon_2 Q_{lo} + \varepsilon_3 Q_{lw} + \varepsilon_4 (Q_{lg})^2 + \varepsilon_5 (Q_{lo})^2 + \varepsilon_6 (Q_{lw})^2 + \varepsilon_7 Q_{lg} Q_{lw} + \varepsilon_8 Q_{lg} Q_{lo} + \varepsilon_9 Q_{lo} Q_{lw} \quad (A4)$$

Where Q represents the flowrate, P_{wh} , the wellhead pressure, $Q_{g,inj}$ the gas lift injection rate, f_{ESP} , the ESP frequency. Subscripts o, w, g, represent the oil, water and gas phase and l represents the pipeline. Coefficients of the proxy models are α , β , δ , and ε respectively.

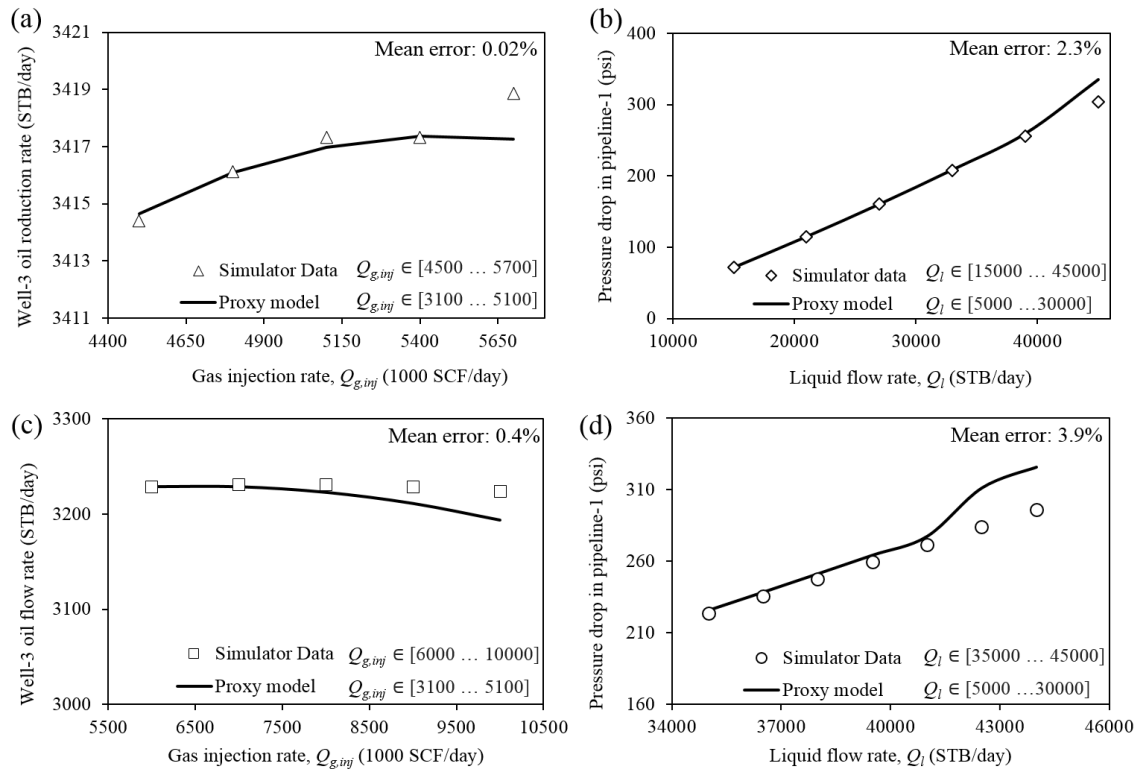


Figure A1: Well and pipeline proxy model validation using PIPESIM® simulation data.

The performance of the implemented proxy models (Eqs. A1-A4) is shown in Figure A1. For brevity, only the proxy models for the gas lift well-3 and pipeline-1 are presented. In performing the validation procedure, a mixture of datasets used in the model development phase and simulation datasets outside this range is applied (Figs. A1a-b). In Figs. A1c-d, the datasets utilised are entirely outside the data range used for the proxy model development. As expected, the proxy model performance with the mixed data set is better than that with the entirely different data range (as reflected in the absolute mean errors). Furthermore, it can be observed from all the plots, that a critical point is reached when the proxy model performance begins to diverge and become inaccurate. This is why an iterative proxy model updating procedure is essential and implemented.

Comparison of heuristic and optimisation methodologies (Kosmidis, 2005)

The economic benefits of mathematical optimisation when applied to oil producing fields was demonstrated by Kosmidis et al. (2005). As reflected in Fig. A2, up to 14 % improvement is obtained. The current work builds upon that of Kosmidis et al. (2005) by incorporating novel elements presented in Section 2 of this paper.

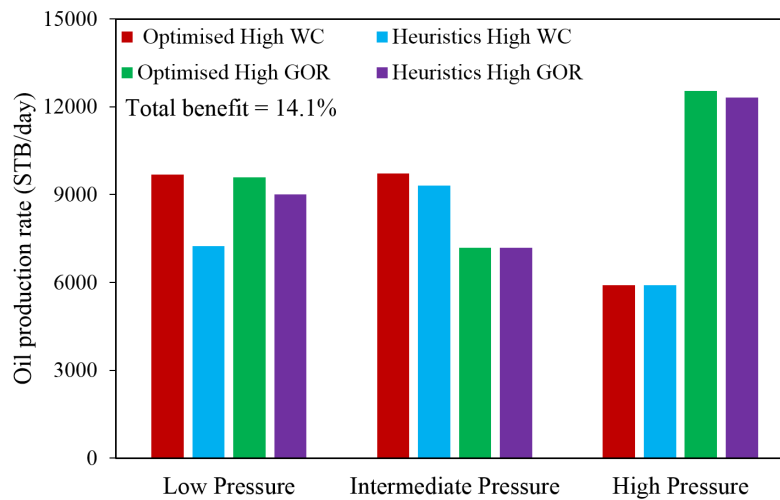


Figure A2: A quantitative comparison of heuristic and optimisation methods for oil production optimisation.

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